# ME 306 Fluid Mechanics II

## Part 4

## Compressible Flow

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## Review of Ideal Gas Thermodynamics

- Ideal gas equation of state is

$$p = \rho R T$$

where $R$ is the gas constant.

- By defining specific volume as $v = 1/\rho$ ideal gas law becomes

$$pv = RT$$

- For an ideal gas internal energy ($\bar{u}$) is a function of temperature only.
- Ideal gas specific heat at constant volume is defined as

$$c_v = \frac{d\bar{u}}{dT}$$

- $c_v$ is also a function of temperature, but for moderate temperature changes it can be taken as constant. In this course we'll take $c_v$ as constant.
- Change in internal energy between two states is (considering constant $c_v$)

$$\bar{u}_2 - \bar{u}_1 = c_v\,(T_2 - T_1)$$



## Review of Ideal Gas Thermodynamics (cont'd)

- Enthalpy is defined as

$$h \;=\; \bar{u} + \frac{p}{\rho} \;=\; \bar{u} + RT$$

- For an ideal gas enthalpy is also a function of temperature only.
- Ideal gas specific heat at constant pressure is defined as

$$c_p = \frac{dh}{dT}$$

- $c_p$ will also be taken as constant in this course. For constant $c_p$ change in enthalpy is

$$h_2 - h_1 = c_p\,(T_2 - T_1)$$

- Combining the definition of $c_v$ and $c_p$

$$c_p - c_v \;=\; \frac{dh}{dT} - \frac{d\bar{u}}{dT} \;=\; R$$



## Review of Ideal Gas Thermodynamics (cont'd)

- For air

$$c_p - c_v = R$$

$$1.005\frac{kJ}{kgK} \qquad 0.718\frac{kJ}{kgK} \qquad 0.287\frac{kJ}{kgK}$$

- Specific heat ratio is (also shown with $\gamma$)

$$k = \frac{c_p}{c_v}$$

which has a value of 1.4 for air.

- Combining the above relations we can also obtain

$$c_p = \frac{Rk}{k-1}\,, \qquad c_v = \frac{R}{k-1}$$

## Review of Ideal Gas Thermodynamics (cont'd)

- Entropy change for an ideal gas is expressed with $Tds$ relations

$$Tds = d\bar{u} + p\, d\left(\frac{1}{\rho}\right) , \qquad Tds = dh - \left(\frac{1}{\rho}\right) dp$$

- Integrating these $Tds$ relations for an ideal gas

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{\rho_1}{\rho_2}\right), \qquad s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

- For an adiabatic (no heat transfer) and frictionless flow, which is known as isentropic flow, entropy remains constant.

**Exercise :** For isentropic flow of an ideal gas with constant specific heat values, derive the following commonly used relations, known as isentropic relations

$$\left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{\rho_2}{\rho_1}\right)^{k} = \left(\frac{p_2}{p_1}\right)$$



## Mach Number and the Speed of Sound

- Compressibility effects become important when a fluid moves with speeds comparable to the local speed of sound ($c$).
- Mach number is the most important nondimensional number for compressible flows

$$Ma = V / c$$

- $Ma < 0.3$ Incompressible flow (density changes are negligible)
- $0.3 < Ma < 0.9$ Subsonic flow (density changes are important, shock waves do not develop)
- $0.9 < Ma < 1.1$ Transonic flow (shock waves may appear and divide the flow field into subsonic and supersonic regions)
- $1.1 < Ma < 5.0$ Supersonic flow (shock waves may appear, there are no subsonic regions)
- $Ma > 5.0$ Hypersonic flow (very strong shock waves and property changes)

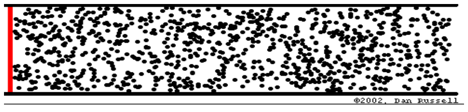


## Speed of Sound ($c$)

- Speed of sound is the rate of propagation of a pressure pulse (wave) of infinitesimal strength through a still medium (a fluid in our case).
- It is a thermodynamic property of the fluid.
- For air at standard conditions, sound moves with a speed of $c = 343$ m/s.

**Exercise:** a) What's the speed of sound in air at 5 km and 10 km altitudes?

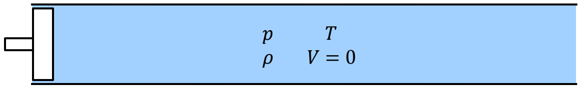
b) What's the speed of sound in water at standard conditions?



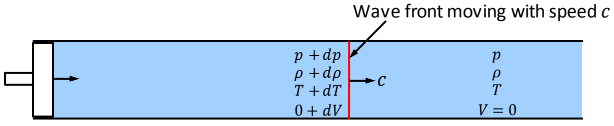



## Speed of Sound (cont'd)

- To obtain a relation for the speed of sound consider the following experiment
- A duct is initially full of still gas with properties $p, \rho, T$ and $V = 0$

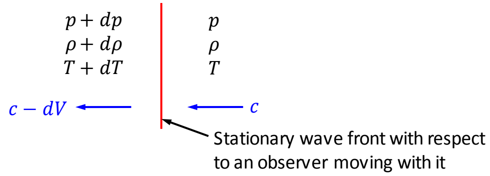



- The piston is pushed into the fluid with an infinitesimal velocity.
- A pressure wave of infinitesimal strength will form and it'll travel in the gas with the speed of sound $c$.
- As it passes over the gas particles it will create infinitesimal property changes.

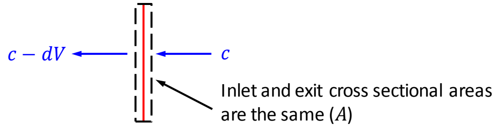


## Speed of Sound (cont'd)

- For an observer moving with the wave front with speed $c$, wave front will be stationary and the fluid on the left and the right would move with relative speeds




- Consider a control volume enclosing the stationary wave front. The flow is one-dimensional and steady. It has one inlet and one exit.






## Speed of Sound (cont'd)

**Exercise:** Using conservation of mass and momentum on the CV of the previous slide, derive the following expression for the speed of sound.

$$c = \sqrt{\left(\frac{dp}{d\rho}\right)_s}$$

Propagation of a sound wave is an isentropic process

**Exercise :** In deriving the speed of sound equation, we did not make use of the energy equation. Show that it can also be used and gives the same result.

**Exercise :** What is the speed of sound for a perfectly incompressible fluid.

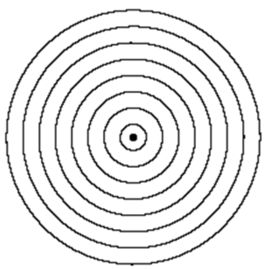
**Exercise :** Show that speed of sound for an ideal gas is given by

$$c = \sqrt{kRT}$$



## Wave Propagation in a Compressible Fluid

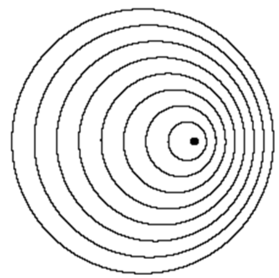
- Consider a point source generating small pressure pulses (sound waves) at regular intervals.
- Case 1 : Stationary source
- Waves travel in all directions symmetrically.
- The same sound frequency will be heard everywhere around the source.





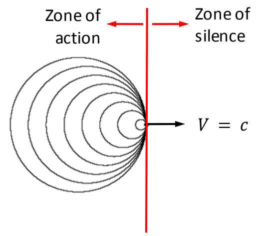
## Wave Propagation in a Compressible Fluid (cont'd)

- Case 2 : Source moving with less than the speed of sound ($Ma < 1$)
- Waves are not symmetric anymore.
- An observer will hear different sound frequencies depending on his/her location.
- This asymmetry is the cause of the Doppler effect.



## Wave Propagation in a Compressible Fluid (cont'd)

- Case 3 : Source moving the speed of sound ($Ma = 1$)
- The source moves with the same speed as the sound waves it generates.
- All waves concentrate on a plane passing through the moving source creating a Mach wave, across which there is a significant pressure change.
- Mach wave separates the field into two as zone of silence and zone of action.

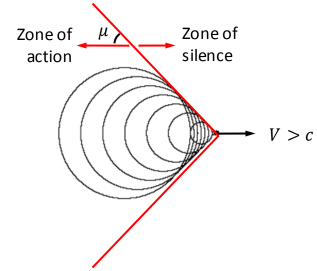



- First aircraft exceeding the speed of sound : http://en.wikipedia.org/wiki/Bell_X-1



## Wave Propagation in a Compressible Fluid (cont'd)

- Case 4 : Source moving with more than the speed of sound ($Ma > 1$)
- The source travels faster than the sound it generates.
- Mach cone divides the field into zones of action and silence.
- Half angle of the Mach cone is called the Mach angle $\mu$.





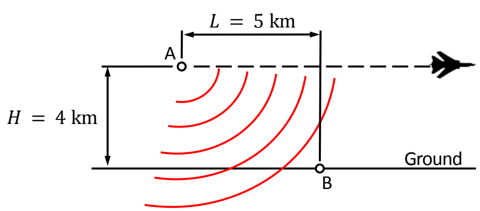

## Wave Propagation in a Compressible Fluid (cont'd)

**Exercise :** For ''Case 4'' described in the previous slide show that

$$\sin(\mu) = 1/Ma$$

**Exercise :** A supersonic airplane is traveling at an altitude of 4 km. The noise generated by the plane at point A reached the observer on the ground at point B after 20 s. Assuming isothermal atmosphere, determine

a) Mach number of the airplane
b) distance traveled by the airplane before the observer hears the noise
c) velocity of the airplane
d) temperature of the atmosphere

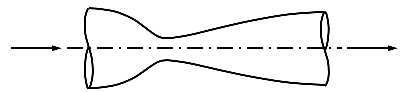



F/A-18 breaking the sound barrier




## 1D, Isentropic, Compressible Flow

- Consider an internal compressible flow, such as the one in a duct of variable cross sectional area



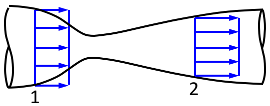
- Flow and fluid properties inside this duct may change due to
  - Cross sectional area change
  - Frictional effects
  - Heat transfer effects

  } NOT the subject of ME 306. Without these the flow is isentropic.

- In ME 306 we'll study these flows as 1D and consider only the effect of area change, i.e. assume isentropic flow.

## 1D, Isentropic, Compressible Flow (cont'd)

- Conservation of energy for a control volume enclosing the fluid between sections 1 and 2 is




$$\underbrace{q - w}_{=0} = \left(h_2 + \frac{V_2^2}{2} + g z_2\right) - \left(h_1 + \frac{V_1^2}{2} + g z_1\right)$$

Heat transfer is zero for adiabatic flow. Also there is no work done other than the flow work.

For gas flows potential energy change is usually negligibly small compared to enthalpy and kinetic energy changes.

- Energy equation becomes

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$



## Stagnation Enthalpy

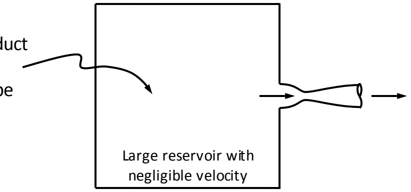
- The sum $\left(h + \frac{V^2}{2}\right)$ is known as stagnation enthalpy and it is constant inside the duct.

$$h_0 = h + \frac{V^2}{2} = \text{constant along the duct}$$

stagnation enthalpy

- It is called ''stagnation'' enthalpy because at a stagnation point velocity is zero and the enthalpy of the gas is equal to $h_0$.

If the fluid is sucked into the duct from a ''large'' reservoir, the reservoir can be assumed to be at the stagnation state.



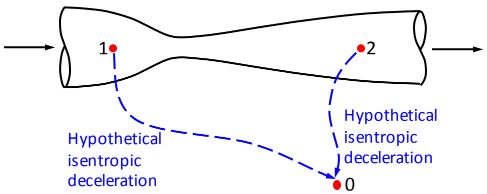



## Stagnation State

- Stagnation state is an important reference state for compressible flow calculations.
- It is the state achieved if a fluid at any other state is brought to rest isentropically.
- For an isentropic flow there will a unique stagnation state.

State 1: $V_1,\ h_1,\ p_1,\ T_1$, etc.

State 2: $V_2,\ h_2,\ p_2,\ T_2$, etc.

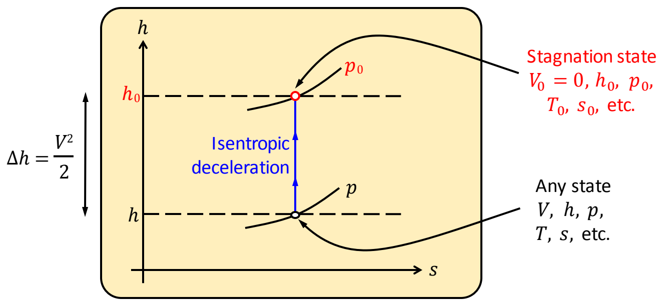



Unique stagnation state: $V_0 = 0,\ h_0,\ p_0,\ T_0$, etc.



## Stagnation State (cont'd)

- Isentropic deceleration can be shown on a $h - s$ diagram as follows




- During isentropic deceleration entropy remains constant.
- Energy conservation: $h_0 + \frac{0^2}{2} = h + \frac{V^2}{2} \quad \rightarrow \quad \Delta h = h_0 - h = \frac{V^2}{2}$

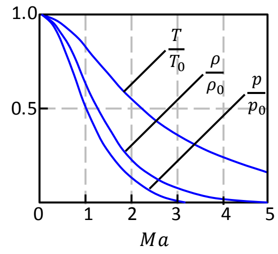
## Stagnation State (cont'd)

**Exercise :** For the isentropic flow of an ideal gas, express the following ratios as a function of Mach number and generate the following plot for air with $k = 1.4$.

$$\frac{T}{T_o} = \left(1 + \frac{k-1}{2} Ma^2\right)^{-1}$$

$$\frac{p}{p_o} = \left(1 + \frac{k-1}{2} Ma^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_o} = \left(1 + \frac{k-1}{2} Ma^2\right)^{-1/(k-1)}$$






## Stagnation State (cont'd)

**Exercise :** An airplane is crusing at a speed of 900 km/h at an altitude of 10 km. Atmospheric air at −60 °C comes to rest at the tip of its pitot tube. Determine the temperature rise of air.

Read about heating of space shuttle during its reentry to the earth's atmosphere.

http://en.wikipedia.org/wiki/Space_Shuttle_thermal_protection_system

**Exercise :** An aircraft cruises at 12 km altitude. A pitot-static tube on the nose of the aircraft measures stagnation and static pressures of 2.6 kPa and 19.4 kPa, respectively. Calculate

a) Mach number of the aircraft

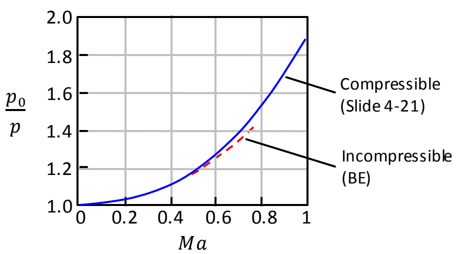
b) speed of the aircraft

c) stagnation temperature that would be sensed by a probe on the aircraft.



## Stagnation State (cont'd)

**Exercise :** Using the incompressible form of the Bernoulli's equation, derive an expression for $p_0/p$ for incompressible flows. Compare it with the one given in Slide 4-21. Determine the Mach number below which two equations agree within engineering accuracy.



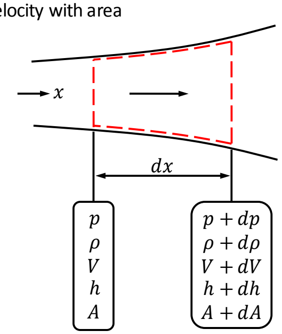



## Simple Area Change Flows (1D Isentropic Flows)

**Exercise :** Consider the differential control volume shown below for 1D, isentropic flow of an ideal gas through a variable area duct. Using conservation of mass, linear momentum and energy, determine the

a) Change of pressure with area

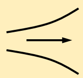
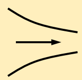
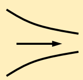
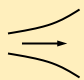
b) Change of velocity with area



## Simple Area Change Flows (cont'd)

- Results of the previous exercise are

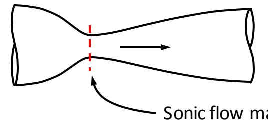
$$\frac{dA}{A} = \frac{dp}{\rho V^2}(1 - Ma^2) \quad , \quad \frac{dA}{A} = -\frac{dV}{V}(1 - Ma^2)$$

| | Subsonic Flow $(Ma < 1)$ | Supersonic Flow $(Ma > 1)$ |
|---|---|---|
| Diffuser<br>$dp > 0$<br>$dV < 0$ |  $dA > 0$ |  $dA < 0$ |
| Nozzle<br>$dp < 0$<br>$dV > 0$ |  $dA < 0$ |  $dA > 0$ |



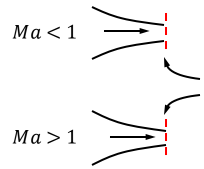
## Simple Area Change Flows (cont'd)

- Sonic flow is a very special case. It can occur
  - when the cross sectional area goes through a minimum, i.e. $dA = 0$



Sonic flow may occur at the throat.

  - or at the exit of a subsonic nozzle or a supersonic diffuser




Sonic flow may occur at these exits.

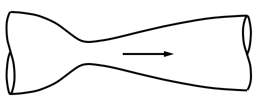


## de Laval Nozzle (C-D Nozzle)

**Exercise :** The nozzle shown below is called a converging diverging nozzle (C-D nozzle or Con-Div nozzle or de Laval nozzle).

Using the table of Slide 4-25 show that it is the only way to

- isentropically accelerate a fluid from subsonic to supersonic speed.
- isentropically decelerate a fluid from supersonic to subsonic speed.



de Laval nozzle



## Critical State

- Critical state is the special state where the Mach number is unity.
- It is a useful reference state, similar to the stagnation state. It is useful even if there is no actual critical state in a flow.
- It is shown with an asterisk, like $T^*$, $p^*$, $\rho^*$, $A^*$, etc.
- Ratios derived in Slide 4-21 can also be written using the critical state.

$$\frac{p_o}{p} = \left(1 + \frac{k-1}{2}Ma^2\right)^{k/(k-1)}$$
$$\frac{T_o}{T} = 1 + \frac{k-1}{2}Ma^2$$
$$\frac{\rho_o}{\rho} = \left(1 + \frac{k-1}{2}Ma^2\right)^{1/(k-1)}$$

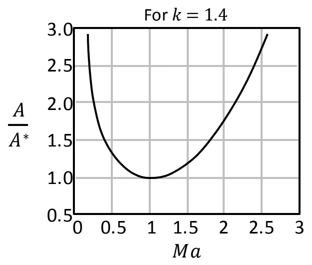
$\xrightarrow{Ma = 1}$

$$\frac{p_o}{p^*} = \left(1 + \frac{k-1}{2}\right)^{k/(k-1)}$$
$$\frac{T_o}{T^*} = 1 + \frac{k-1}{2}$$
$$\frac{\rho_o}{\rho^*} = \left(1 + \frac{k-1}{2}\right)^{1/(k-1)}$$

## Critical State (cont'd)

**Exercise :** Similar to the ratios given in the previous slide, following area ratio is also a function of $Ma$ and $k$ only. Derive it.

$$\frac{A}{A^*} = \frac{1}{Ma}\left(\frac{1+\frac{k-1}{2}Ma^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$




**Exercise :** Derive an expression in terms of $Ma$ and $k$ for the following non-dimensional mass flow rate.

$$\frac{\dot{m}\sqrt{RT_0}}{Ap_0}$$



## Isentropic Flow Table

- It provides the following ratios at different Mach numbers for a fixed $k$ value.

$$\frac{T}{T_0} \quad \frac{p}{p_0} \quad \frac{\rho}{\rho_0} \quad \frac{A}{A^*} \quad \frac{\dot{m}\sqrt{RT_0}}{Ap_0}$$

**APPENDIX C.3**

**ISENTROPIC FLOW OF A PERFECT GAS ($k = 1.4$)**

| $M$ | $\frac{p}{p_0}$ | $\frac{T}{T_0}$ | $\frac{\rho}{\rho_0}$ | $\frac{A}{A^*}$ | $\frac{I}{I^*}$ | $\frac{\dot{m}\sqrt{RT_0}}{Ap_0}$ |
|---|---|---|---|---|---|---|
| 0.00 | 1.0000 | 1.0000 | 1.0000 | ∞ | ∞ | 0.00000 |
| 0.01 | 0.9999 | 1.0000 | 1.0000 | 57.87 | 45.65 | 0.01183 |
| 0.02 | 0.9997 | 0.9999 | 0.9998 | 28.94 | 22.83 | 0.02366 |
| 0.03 | 0.9994 | 0.9998 | 0.9996 | 19.30 | 15.23 | 0.03548 |
| 0.04 | 0.9989 | 0.9997 | 0.9992 | 14.48 | 11.43 | 0.04728 |
| 0.05 | 0.9983 | 0.9995 | 0.9988 | 11.59 | 9.158 | 0.05907 |
| 0.06 | 0.9975 | 0.9993 | 0.9982 | 9.666 | 7.643 | 0.07084 |
| 0.07 | 0.9966 | 0.9990 | 0.9976 | 8.292 | 6.562 | 0.08258 |

Aksel's Fluid Mechanics textbook



## Exercises for Simple Area Change Flows

**Exercise :** A converging duct is fed with air from a large reservoir where the temperature and pressure are 350 K and 200 kPa. At the exit of the duct, cross-sectional area is 0.002 m$^2$ and Mach number is 0.5. Assuming isentropic flow

a) Determine the pressure, temperature and velocity at the exit.

b) Find the mass flow rate.

**Exercise :** Air is flowing isentropically in a diverging duct. At the inlet of the duct, pressure, temperature and velocity are 40 kPa, 220 K and 500 m/s, respectively. Inlet and exit areas are 0.002 m$^2$ and 0.003 m$^2$.

a) Determine the Mach number, pressure and temperature at the exit.

b) Find the mass flow rate.



## Exercises for Simple Area Change Flows (cont'd)

**Exercise :** (Fox) Air flows isentropically in a channel. At an upstream section 1, Mach number is 0.3, area is 0.001 m$^2$, pressure is 650 kPa and temperature is 62 °C. At a downstream section 2, Mach number is 0.8.

a) Sketch the channel shape.

b) Evaluate properties at section 2.

c) Plot the process between sections 1 and 2 on a $T-s$ diagram.

## Shock Waves

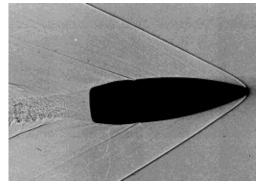
- Sound wave is a weak wave, i.e. property changes across it are infinitesimally small.
- $\Delta p$ across a sound wave is in the order of $10^{-9} - 10^{-3}$ atm.

- Shock wave is a strong wave, i.e. property changes across it are finite.
- Shock waves are very thin, in the order of $10^{-7}$ m.
- Fluid particles decelerate with millions of $g$'s through a shock wave.
- Shock waves can be stationary or moving.
- They can be normal (perpendicular to the flow direction) or oblique (inclined to the flow direction).
- A shock wave can be thought as a tool used for a flow to adjust itself to downstream conditions.

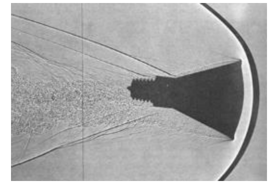
- In ME 306 we'll consider stationary normal shock waves for 1D flows inside ducts.



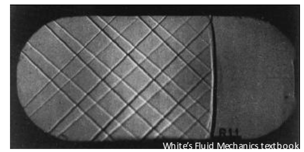
## Shock Waves (cont'd)



Oblique shock wave attached to the sharp nose of a bullet moving at supersonic speed.



Detached curved shock wave ahead of a blunt nosed object moving at supersonic speed.



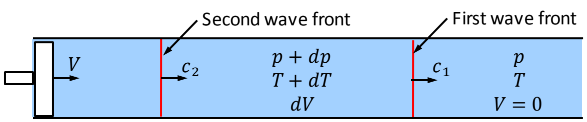

Normal shock wave in a supersonic nozzle. Flow is from left to right. Extra waves are due to surface roughness



## Formation of a Strong Wave

- A strong wave is formed by the accumulation of several weak compression waves.
- Compression waves are the ones across which pressure increase and velocity decrease in the flow direction.
- Sound wave is a weak compression wave.

- Consider a piston pushed with a finite velocity $V$ in a cylinder filled with still gas.
- We can decompose piston's motion into a series of infinitesimally small disturbances.
- Weak compression waves will emerge from the piston, one after the other.
- The first two of such waves are sketched below.






## Formation of a Strong Wave (cont'd)

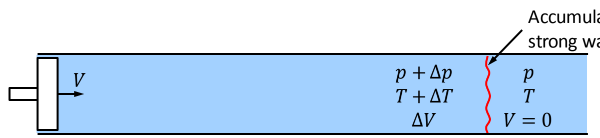
- First wave will cause an increase in temperature behind it.
- Second wave will move faster and eventually may catch the first one.

$$c_2 > c_1$$

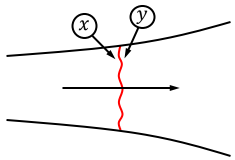
- A third one, which is not shown, will move even faster and may catch the first two waves.

$$c_3 > c_2 > c_1$$

- Weak compression waves have a chance to accumulate into a strong wave of finite strength.
- Weak expansion waves that'll be generated by pulling the piston to the left will not form such a strong wave.



## Normal Shock Wave




- Consider a stationary normal shock wave in a duct of variable cross sectional area.
- Upstream and downstream states are denoted by $x$ and $y$.
- Due to very sudden, finite property changes, the process across the wave is considered to be non-isentropic. But it is considered to be adiabatic.
- There are two different stagnation states, state $0x$ for the flow before the shock and state $0y$ for the flow after the shock.

$$p_{0x} \neq p_{0y} \quad \text{and} \quad \rho_{0x} \neq \rho_{0y}$$

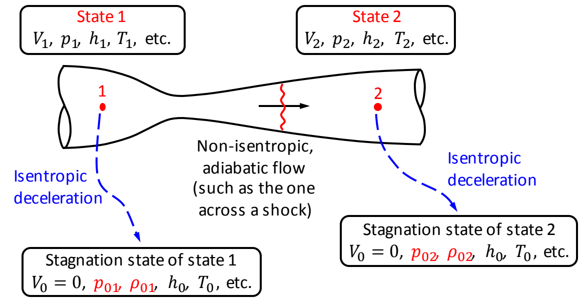
- However, because of the adiabatic assumption, stagnation temperatures and enthalpies are the same.

$$T_{0x} = T_{0y} = T_0 \quad \text{and} \quad h_{0x} = h_{0y} = h_0$$



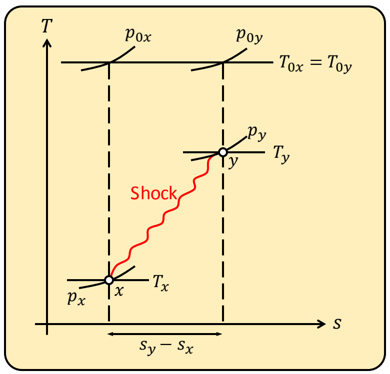
## Stagnation State of a Non-isentropic, Adiabatic Flow

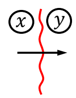
- Stagnation state concept can also be used for non-isentropic flows, but there will be multiple such states.
- If the flow is adiabatic $h_0$ , $T_0$ and $c_0$ will be unique, but not other stagnation properties such as $p_0$ or $\rho_0$.






## Shock Wave on the $T - s$ Plane



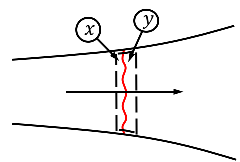




- A similar one can be drawn on the $h - s$ plane.



## Property Changes Across a Shock Wave

- Governing equations for the 1D flow inside the control volume enclosing the shock wave are




- Continuity : $|\dot{m}| = \rho_x V_x A = \rho_y V_y A$ where $A = A_x = A_y$
- Momentum : $(p_x - p_y) A = |\dot{m}| (V_y - V_x)$
- Energy : $h_0 = h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2}$
- Second Law : $s_y > s_x$

## Property Changes Across a Shock Wave (cont'd)

- For the flow of an ideal gas with constant specific heats, we can work on the equations of the previous slide to get the following (study the details from text books)
- Donwstream Mach number : $Ma_y = \sqrt{\dfrac{(k-1)Ma_x^2 + 2}{2kMa_x^2 - (k-1)}}$
- Temperature ratio : $\dfrac{T_y}{T_x} = \dfrac{\left(1 + \frac{k-1}{2}Ma_x^2\right)\left(\frac{2k}{k-1}Ma_x^2 - 1\right)}{\frac{(k+1)^2}{2(k-1)}Ma_x^2}$
- Pressure ratio : $\dfrac{p_y}{p_x} = \dfrac{2k}{k+1}Ma_x^2 - \dfrac{k-1}{k+1}$
- Density ratio : $\dfrac{\rho_y}{\rho_x} = \dfrac{(k+1)Ma_x^2}{2 + (k-1)Ma_x^2}$
- Velocity ratio : $\dfrac{V_y}{V_x} = \dfrac{\rho_x}{\rho_y}$



## Property Changes Across a Shock Wave (cont'd)

- Stagnation pressure ratio : $\dfrac{p_{0y}}{p_{0x}} = \left(\dfrac{\frac{k+1}{2}Ma_x^2}{1 + \frac{k-1}{2}Ma_x^2}\right)^{\frac{k}{k-1}} \left(\dfrac{2k}{k+1}Ma_x^2 - \dfrac{k-1}{k+1}\right)^{\frac{1}{1-k}}$
- Critical area ratio : $\dfrac{A_y^*}{A_x^*} = \dfrac{p_{0x}}{p_{0y}}$
- Entropy change : $\dfrac{s_y - s_x}{R} = -ln\dfrac{p_{0y}}{p_{0x}}$
- These relations are functions of $Ma_x$ and $k$ only. They are usually plotted or tabulated.
- Flow before the shock and after the shock are isentropic, but not across the shock.
- $T_0$ is the same before and after the shock, due to adiabatic assumption ($T_{0x} = T_{0y}$).
- $p_0$ changes across the shock ($p_{0x} \neq p_{0y}$).
- Critical states before and after the shock are different. This is seen from $A_x^* \neq A_y^*$)



## Property Changes Across a Shock Wave (cont'd)

- Tabulated form of these normal shock relations look like the following

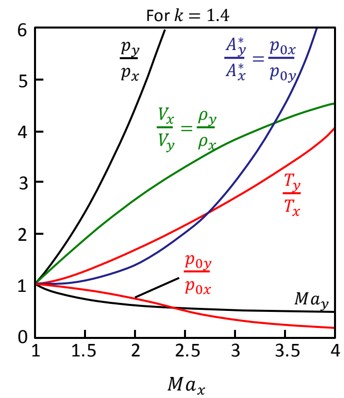
**APPENDIX C.4**
**FLOW OF A PERFECT GAS ACROSS A NORMAL SHOCK WAVE (k = 1.4)**

| $M_x$ | $M_y$ | $\frac{p_y}{p_x}$ | $\frac{T_y}{T_x}$ | $\frac{\rho_y}{\rho_x} = \frac{V_x}{V_y}$ | $\frac{p_{0y}}{p_{0x}} = \frac{A_x^*}{A_y^*}$ | $\frac{s_y - s_x}{R}$ |
|---|---|---|---|---|---|---|
| 1.00 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.00000000 |
| 1.01 | 0.9901 | 1.023 | 1.007 | 1.017 | 1.0000 | 0.00000127 |
| 1.02 | 0.9805 | 1.047 | 1.013 | 1.033 | 1.0000 | 0.00000997 |
| 1.03 | 0.9712 | 1.071 | 1.020 | 1.050 | 1.0000 | 0.00003299 |
| 1.04 | 0.9620 | 1.095 | 1.026 | 1.067 | 0.9999 | 0.00007672 |
| 1.05 | 0.9531 | 1.120 | 1.033 | 1.084 | 0.9999 | 0.0001470 |
| 1.06 | 0.9444 | 1.144 | 1.039 | 1.101 | 0.9998 | 0.0002493 |
| 1.07 | 0.9360 | 1.169 | 1.046 | 1.118 | 0.9996 | 0.0003886 |

Aksel's book



## Property Changes Across a Shock Wave (cont'd)



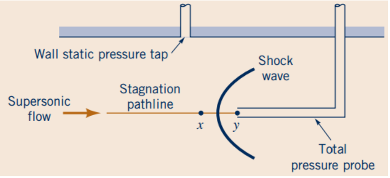

- Across a normal shock wave

| | |
|---|---|
| $Ma,\ V,\ p_0$ | decreases |
| $p,\ T,\ \rho,\ A^*,\ s$ | increases |
| $T_0$ | remains the same |

- Kinetic energy of the fluid after the shock wave is smaller than the one that would be obtained by a reversible compression between the same pressure limits.
- Lost kinetic energy is the reason of temperature increase across the shock wave.

## Normal Shock Wave (cont'd)

**Exercise :** (Munson) A total pressure probe is inserted into a supersonic air flow. A shock wave forms just upstream of the stagnation point. The probe measures a stagnation pressure of 410 kPa. The stagnation temperature at the probe tip is measured with a thermocouple and found to be 555 K. The static pressure upstream of the shock is measured with a wall tap to be 83 kPa. Determine the Mach number and the velocity of the flow.






## Normal Shock Wave (cont'd)

**Exercise :** (Fox) Air with speed 668 m/s, temperature 5 ºC and pressure 65 kPa goes through a normal shock wave.

a) Determine the Mach number, pressure, temperature, speed, stagnation pressure and stagnation temperature after the shock wave,

b) Calculate the entropy change across the shock wave.

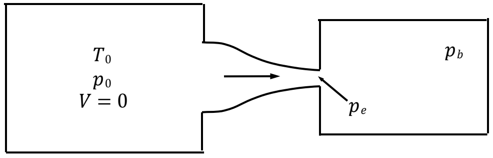
c) Show the process on a $T - s$ diagram.

**Exercise :** Supersonic air flow inside a diverging duct is slowed down by a normal shock wave. Mach number at the inlet and exit of the duct are 2.0 and 0.3. Ratio of the exit to inlet cross sectional areas is 2. Pressure at the inlet of the duct is 40 kPa. Assuming adiabatic flow determine the pressure after the shock wave and at the exit of the duct.



## Operation of a Converging Nozzle

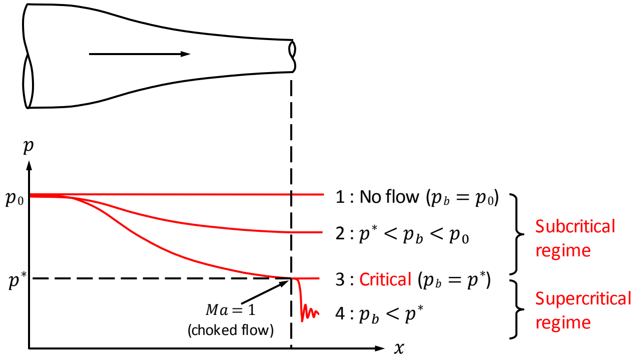
- Consider a converging nozzle.
- Gas is provided by a large reservoir with stagnation properties, $T_0$ and $p_0$.
- Back pressure $p_b$ is adjusted using a vacuum pump to obtain different flow conditions inside the nozzle.
- Exit pressure $p_e$ and back pressure $p_b$ can be equal or different.






## Operation of a Converging Nozzle (cont'd)

- First set $p_b = p_0$. There will be no flow.
- Gradually decrease $p_b$. Following pressure distributions will be observed.



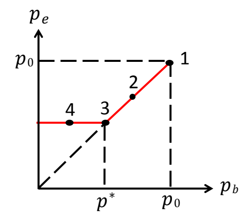
## Operation of a Converging Nozzle (cont'd)

- When air is supplied from a stagnation reservoir, flow inside a converging nozzle always remains subsonic.
- For the subcritical regime, mass flow rate increases as $p_b$ decreases.
- State shown with * is the critical state. When $p_b$ is lowered to the critical value $p^*$, exit Mach number reaches to 1 and flow is said to be choked.
- If $p_b$ is lowered further, flow remains choked. Pressure and Mach number at the exit do not change. Mass flow rate through the nozzle does not change.
- For $p_b < p^*$, gas exits the nozzle as a supercritical jet with $p_e > p_b$. Exit jet undergoes a non-isentropic expansion to reduce its pressure to $p_b$.
- From slide 4-28 $\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$. For air ($k = 1.4$) choking occurs when $\frac{p^*}{p_0} = 0.528$.

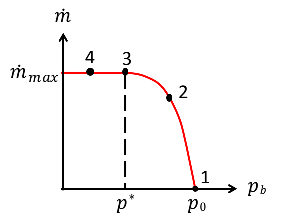


## Operation of a Converging Nozzle (cont'd)

Variation of $p_e$ with $p_b$




Variation of $\dot{m}$ with $p_b$




- Case 1 is the no flow case.
- From case 1 to case 3, lowering $p_b$ decreases $p_e$ and increases $\dot{m}$.
- Case 3 is the critical case with minimum possible $p_e$ and maximum possible $\dot{m}$.
- Cases 3 and 4 are choked flow with $Ma_e = 1$.



## Operation of a Converging Nozzle (cont'd)

**Exercise :** (Aksel's book) A converging nozzle is fed with air from a large reservoir where the pressure and temperature are 150 kPa and 300 K. The nozzle has an exit cross sectional area of 0.002 m². Back pressure is set to 100 kPa. Determine

a) the pressure, Mach number and temperature at the exit.

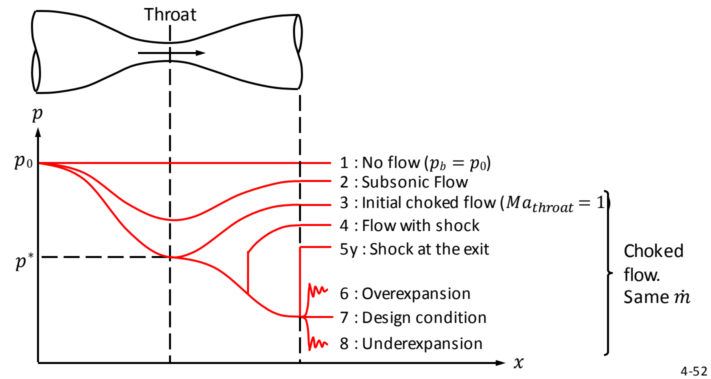
b) mass flow rate through the nozzle.

**Exercise :** (Aksel's book) Air flows through a converging duct and discharges into a region where the pressure is 100 kPa. At the inlet of the duct, the pressure, temperature and speed are 200 kPa, 290 K and 200 m/s. Determine the Mach number, pressure and temperature at the nozzle exit. Also find the mass flow rate per unit area.



## Operation of a Conv-Div Nozzle

- We first set $p_b = p_0$ and then gradually decrease $p_b$.

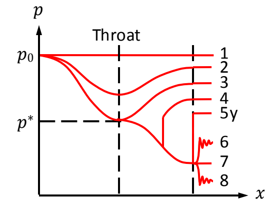


## Operation of a Conv-Div Nozzle (cont'd)

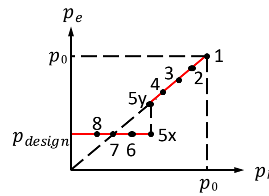
- Flow inside the converging section is always subsonic.
- At the throat the flow can be subsonic or sonic.
- The flow is choked if $Ma_{\text{throat}} = 1$. This corresponds to the maximum $\dot{m}$ that can pass through the nozzle.
- Under choked conditions the flow in the diverging part can be subsonic (case 3) or supersonic (cases 6, 7 ,8).
- Depending on $p_b$ there may be a shock wave in the diverging part. Location of the shock wave is determined by $p_b$.
- Design condition corresponds to the choked flow with supersonic exit without a shock.
- Overexpansion ($p_e < p_b$): Exiting jet finds itself in a higher pressure medium and contracts. Underexpansion ($p_e > p_b$): Exiting jet finds itself in a lower pressure medium and expands. For details and pictures visit http://aerorocket.com/Nozzle/Nozzle.html and http://www.aerospaceweb.org/question/propulsion/q0224.shtml



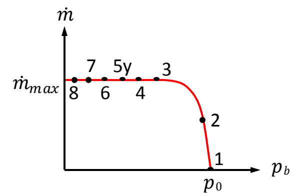
## Operation of a Conv-Div Nozzle (cont'd)




Variation of $p_e$ with $p_b$

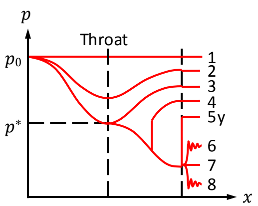


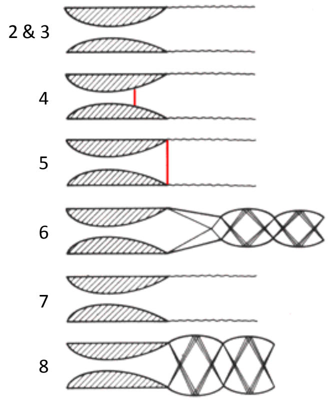

Variation of $\dot{m}$ with $p_b$






## Operation of a Conv-Div Nozzle (cont'd)




http://aerorocket.com

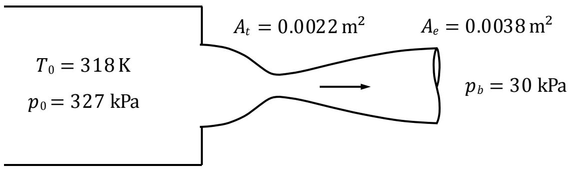


## Operation of a Conv-Div Nozzle (cont'd)

**Exercise :** Air is supplied to a C-D nozzle from a large reservoir where stagnation pressure and temperature are known. Determine

a) the Mach number, pressure and temperature at the exit

b) the mass flow rate



# Operation of a Conv-Div Nozzle (cont'd)

**Exercise :** Air flows in a Conv-Div nozzle with an exit to throat area ratio of 2.1. Properties at a section in the converging part are measured as follows.

a) Determine the ranges of back pressure for different flow regimes.

b) If a shock wave is observed where the area is twice of the throat area, determine the back pressure.

$p_1 = 128\,\text{kPa}$

$T_1 = 294\,\text{K}$

$V_1 = 135\,\text{m/s}$